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A modified transfer matrix method for prediction of transmission loss of multilayer acoustic materials

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A B S T R A C T

A new experimental approach, herein referred to as a modified transfer matrix method, for evaluating normal incidence sound transmission loss of multilayer solid materials, is presented. As it is the key element for predicting the transmission loss of a material, the original transfer matrix was measured directly via a standing wave tube method. During predictive studies of multilayer materials, the transfer matrices of solid layers were modified in accord with data from the vibration of thin plates and the mass law effect. We validated the feasibility of this method through experiments on several different kinds of materials. This method can be applied in designing multilayer sound insulation materials that are widely used in automotive engineering and building acoustics.

1. Introduction

Multilayer materials have been widely used as an effective sound attenuation material in areas including automotive engineering and building acoustics. Thus proper design and optimization of the acoustical properties of multilayer materials have become an attractive subject, and many theories have been developed [1–4]. In real applications, in order to know the acoustical properties of a certain multilayer material easily and rapidly, prediction is often needed instead of direct measurement. In most cases, feasible prediction via a so-called transfer matrix method is often used [5,6]. This method is based on a theory, which says that the relation between the pressure and bulk flow of two ends of a sound propagating route can be expressed by a matrix. The transfer matrix can be looked upon as an inherent property of a material because of its invariance. If the transfer matrix of a material is known, most of the acoustical properties of the material can be obtained. Because of the continuity of sound pressure and velocity, the matrices of all the component layers can be combined together into a total matrix which can be used to predict the acoustical properties of multilayer materials. The type of transfer matrix is determined by such factors as physical properties of the material, boundary conditions, selected state variables and so on. To determine an appropriate type of transfer matrix, an accurate model of sound propagation must be established. Although different types of materials are involved, this work aims to model normal incidence sound propagation with the same type of matrix.

To measure the acoustical properties of a certain material, the reverberation room method [7] has been commonly used. Although this method provides authoritative test results, it is not easy to use. Assurance with this method not only takes time, but also requires rare types of equipment such as reverberation rooms, high quality microphones and large specimens. As another option to measure the normal incident acoustical properties of materials, the standing wave tube

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method has also been used widely and adopted as an ISO standard [8]. In this research, the standing wave tube method is used to measure the transfer matrix of materials for further prediction.

Because the measurable frequency range is determined by the diameter of the tube, when the standing wave tube method is adopted, it is necessary to use two sets of tubes which have different widths and lengths to produce a result in the desired frequency range. For sound absorbing materials, the values of the absorption ratio measured with low (125–1600 Hz) and high (500–6400 Hz) frequency range tubes, respectively, match well on the overlapped frequency range. But for sound insulation materials, because of the phenomena of resonance of the specimen, the measured values of transmission loss in low and high frequency ranges do not match well. In order to overcome this shortage to get a useful full range result, in this research, a so-called mass law effect [9] is adopted in order to connect the measured values of transmission loss in low and high frequency ranges. The mass law is a theoretical rule which is applied to most limp and unbounded panels in a certain range of frequency. It indicates that the most important physical property, which controls the airborne sound transmission loss, is the mass per unit area of its component layers. The mass law equation predicts that each time the frequency of measurement or the mass per unit area of a single layer wall is doubled, the transmission loss increases about 6 dB. Three types of mass law have been commonly used for the cases of normal, field and random sound incidence, and there are certain relationships among them. For the specimens clamped inside the standing wave tube, the mass law effect also exists in a certain range of frequency. This effect can be shown clearly by connecting the values of transmission loss measured with low and high frequency standing wave tubes using the corresponding mass law controlled frequency range. This connected curve shows the normal incidence mass law effect. It can be used to predict the field incidence mass law effect which can only be obtained with the reverberation room method.

In an earlier article, it was shown that a transfer matrix based prediction method can be applied to estimate the normal incidence acoustical properties of multilayered materials in a standing wave duct [10]. Either a two-cavity method (TCM) or two-load method (TLM) was suggested to build the transfer matrix, which can subsequently be used to calculate some essential acoustical properties in the high frequency range. Both TCM and TLM have some disadvantages which limit their applicability for multiple types of materials. Although a hybrid method which integrates both of these two methods has been suggested, the procedure was still faulty. In this work, it is aimed to build the transfer matrices of specimens in another, more appropriate way in both low and high frequency ranges.

The purpose of this research is to predict the acoustical properties of multilayer materials, especially sound transmission loss, by using the corresponding transfer matrices, which are originally measured with the standing wave tube method and then modified with plate theory and mass law effects. There are four steps during the prediction: at first, the original transfer matrices which can represent the acoustical properties should be measured with both low and high frequency standing wave tubes. Then these two matrices need to be modified with plate theory for multilayer prediction. The two modified transfer matrices, which relate to the same material but different frequency stages, need to be modified into a whole matrix by referring to the mass law effect. Finally, the retrieved transfer matrix is used as a component element to predict the acoustical properties (transmission loss) of a multilayer material in a wide range of frequencies.

2. Preparation for the experiment

The experiments in this paper were made by using two sets of standing wave tubes (Acoustic Duct of SCIEN Co.) to measure the acoustical properties in the low (125–1600 Hz) and high (500–6400 Hz) frequency ranges. A PC program was also used for the FFT analysis and for the calculation of the transfer matrix. The setup of this experiment is shown in Fig. 1, where A, B, C and D represent the complex amplitudes of positive- and negative going plane waves in the up- and downstream segments of the standing wave tube. The result of the measurement is calculated by using a four-microphone standing wave ratio method (SWRM), which is described in Appendix A.

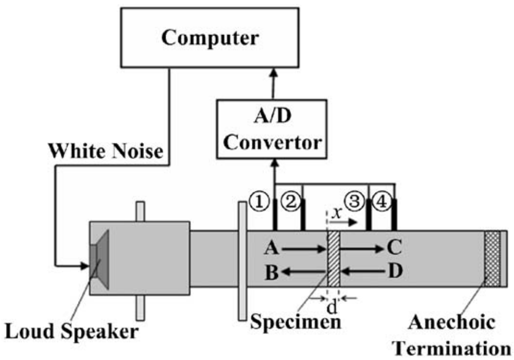

Fig. 1. Setup of the experiment.

Table 1
Material properties of the tested specimens.

No.	Material ingredient	Physical property	Thickness (mm)	Area density (kg/m^2)
1	Rigid foam	Porous	20	0.65
2	Foam	Porous	18	0.46
3	PVC synthetic leather	Solid	3	0.78
4	Electrostatic rubber	Solid	1.2	2.3
5	Synthetic rubber	Solid	1	2.4
6	Synthetic rubber	Solid	1.5	3.4

The specimens used in the experiments are presented in Table 1. There are four solid specimens which are commonly used in sound insulation, and two porous specimens which can be used as cores of sandwich panels. The multilayer materials used in the experiments were assembled with these single layer specimens.

3. Original transfer matrix

Different kinds of materials have different forms of transfer matrices. For fluid materials, the inside acoustic field is determined by the parameters of sound pressure and particle velocity. So a 2×2 matrix is applied as the transfer matrix to relate these two parameters at two sides of this material. For elastic solid acoustical materials, the difference from the fluid case is that two kinds of waves, such as rotational and dilatational, can propagate inside the medium. So four parameters are needed to represent the acoustic field, and the transfer matrix becomes a 4×4 matrix [11]. For porous materials, three different kinds of waves exist inside the medium, so the transfer matrix is a 6×6 matrix [12]. In spite of these classifications, a 2×2 transfer matrix is found to be reasonable to represent any type of single layer material clamped inside standing wave tubes [13].

In standing wave tubes, the acoustic field is assumed to be composed of a series of plane waves propagating in the axial direction. For the specimen clamped inside the tube, adopting sound pressure P and particle velocities V as the two state variables, the following matrix relation can be written to relate state variables on the two surfaces of the specimen.

$$\begin{bmatrix} P \\ V \end{bmatrix}_{x=0} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} P \\ V \end{bmatrix}_{x=d} \tag{1}$$

Using the four elements T_{11}, T_{12}, T_{21} and T_{22} of the transfer matrix, one can calculate the absorption ratio and the transmission loss directly as in [14]:

Absorption ratio:

$$\alpha = 1 - \left|\frac{T_{11} - Z_0 T_{21}}{T_{11} + Z_0 T_{21}}\right|^2 \tag{2}$$

Transmission loss:

$$TL = 20\lg\left(\frac{1}{2}\left|T_{11} + \frac{T_{12}}{Z_0} + Z_0 T_{21} + T_{22}\right|\right) \tag{3}$$

where Z_0 is the characteristic impedance of the ambient medium.

To calculate the transfer matrix in this equation, the values of P and V are needed. In the standing wave tubes, four microphones are needed to measure the pressure of incident, transmitted and reflected sound waves. The particle velocity can be calculated by dividing the sound pressure with the characteristic impedance of the air.

Most of the sound insulation materials are isotropic, and it was already proved that for isotropic materials, the transfer matrix has properties of reciprocity and symmetry [15]:

$$T_{11} = T_{22} \tag{4}$$

$$T_{11}T_{22} - T_{12}T_{21} = 1 \tag{5}$$

After some derivations, the four elements of this transfer matrix can be calculated with the following equations:

$$T_{11} = \frac{P_{x=d}V_{x=d} + P_{x=0}V_{x=0}}{P_{x=0}V_{x=d} + P_{x=d}V_{x=0}} = \frac{A^2 - B^2 + C^2\,e^{-2jkd} - D^2\,e^{2jkd}}{2AC\,e^{-jkd} - 2BD\,e^{jkd}} \tag{6}$$

$$T_{12} = \frac{P^2_{x=0} - P^2_{x=d}}{P_{x=0}V_{x=d} + P_{x=d}V_{x=0}} = \frac{\rho_0 c[A^2 + B^2 - C^2\,e^{-2jkd} - D^2\,e^{2jkd} + 2(AB - CD)]}{2AC\,e^{-jkd} - 2BD\,e^{jkd}} \tag{7}$$

$$T_{21}=\frac{V_{x=0}^2-V_{x=d}^2}{P_{x=0}V_{x=d}+P_{x=d}V_{x=0}}=\frac{A^2+B^2-C^2\,e^{-2jkd}-D^2\,e^{2jkd}-2(AB+CD)}{\rho_0 c(2AC\,e^{-jkd}-2BD\,e^{jkd})} \tag{8}$$

$$T_{22}=T_{11} \tag{9}$$

where k is the wavenumber in the ambient fluid, ρ_0 the ambient fluid density and c the ambient sound speed.

This measured transfer matrix can be used to obtain the transmission loss of a single layer material, but for multilayer materials which are mostly used in real applications, different situations should be discussed. Sometimes, the thickness of the multilayer specimen is too thick to measure it in standing wave tubes directly. In this case, its acoustic properties can be estimated by the total transfer matrix which combines the transfer matrices of each layer. Fig. 2 shows a random layered multilayer material in which component layers' transfer matrices are $\mathbf{TM_1}$, $\mathbf{TM_2}$, and $\mathbf{TM_3}$ up to $\mathbf{TM_n}$. The total transfer matrix of this multilayer material is the product of all the layers' matrices, and the sequence of each matrix should be the same as the sequence of the corresponding layer.

If all of the layers are porous or perforated, each layer can be looked upon as a combination of the fluid phase and solid phase. If the fluid component of a layer is much greater than the solid component, it can be seen as an isolated system in which the internal acoustic field is not affected by other layers. Considering the continuity of sound pressure and velocity, the total transfer matrix can be estimated by multiplying all the matrices together, and the sequence of each matrix should be the same as the sequence of the corresponding layer

$$\mathbf{TM_{Total}}=\mathbf{TM_1}(porous)\times\mathbf{TM_2}(porous)\times\mathbf{TM_3}(porous)\cdots\mathbf{TM_n}(porous) \tag{10}$$

For porous materials, transmission loss is often very low, and the curves in the low and high frequency ranges match well, so there is no need to modify the transfer matrix. The transmission loss of a multilayer porous material can be predicted by substituting Eq. (10) into Eq. (3). Fig. 3 shows the measured and predicted result of an assembled two layer porous specimen (Specimens 1+2).

If the layers are solid, because all of the layers are linked together, the acoustic properties of each layer are affected by others, so the total matrix cannot be calculated by just multiplying the transfer matrix of each layer measured with the standing wave tubes. Unlike multilayer porous materials, if every layer is very thin, the particle velocities at the boundary of every layer ought to be the same in multilayer solid materials, which means the measured transfer matrix should be revised.

4. Transfer matrix with bending effect

The thin plate theory is applied here to modify the transfer matrix of a solid material, and this modified transfer matrix is intended to predict the acoustical properties of a multilayer solid material.

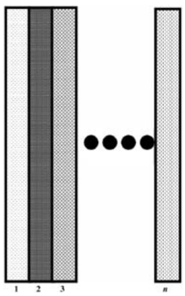

Fig. 2. A random layered material.

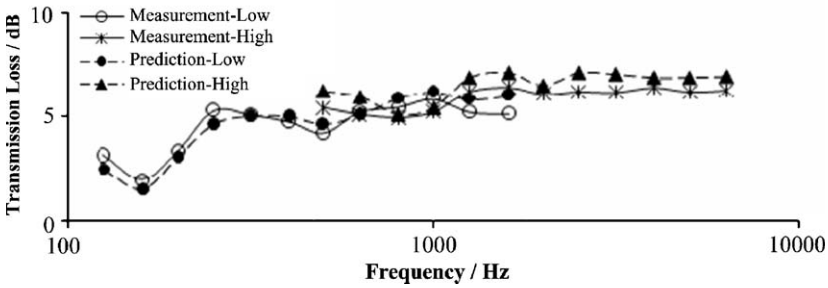

Fig. 3. Measured transmission loss of Specimens 1+2.

Analysis will be limited to the symmetric vibration of a clamped, isotropic and circular plate. Following the thin plate theory, the Euler–Bernoulli differential equation that governs the bending vibration of such a plate is [16]:

$$P_i - P_t = D\frac{\partial^4 w}{\partial x^4} + 2D\frac{\partial^4 w}{\partial x^2 \partial y^2} + D\frac{\partial^4 w}{\partial y^4} + \rho_s\frac{\partial^2 w}{\partial t^2} \tag{11}$$

where P_i and P_t are the sound pressures on the two surfaces of the plate, D the flexural rigidity of the specimen, w the displacement of the plate in the vertical direction, and ρ_s the area density of the material.

Assume periodic vibration,

$$w(x,y,t) = W(x,y)e^{j\omega t} \tag{12}$$

Referring [17], the equation of motion, which must satisfy $W(x, y)$ is

$$\nabla^4 W - k^4 W = 0 \tag{13}$$

where k is the eigenvalue under consideration.

It is assumed that the plate is thin enough, thus the particle velocities on both sides should be the same.

$$V_i = V_t = \frac{\partial w(x,y,t)}{\partial t} = j\omega W(x,y)e^{j\omega t} \tag{14}$$

Substitution of Eq. (12) into Eq. (11) yields

$$P_i - P_t = D\,e^{j\omega t}\nabla^4 W - \rho_s\omega^2 W(x,y)e^{j\omega t} = \frac{1}{j\omega}\left(D\frac{\nabla^4 W}{W} - \rho_s\omega^2\right)V_t = \frac{1}{j\omega}(Dk^4 - \rho_s\omega^2)V_t \tag{15}$$

Eqs. (14,15) can be written together in the form of a matrix

$$\begin{bmatrix} P \\ V \end{bmatrix}_i = \begin{bmatrix} 1 & \frac{1}{j\omega}(Dk^4 - \rho_s\omega^2) \\ 0 & 1 \end{bmatrix}\begin{bmatrix} P \\ V \end{bmatrix}_t \tag{16}$$

Eq. (16) shows an approximate relation between the acoustical parameters on both sides of elastic solid specimens clamped inside the standing wave tubes. In the transfer matrix derived from Eq. (16), only the element T_{12} is unfixed. Direct calculation can be applied to obtain the value of T_{12}, but it is very complicated and time consuming. As another approach to obtain T_{12}, a modification on the transfer matrix measured with the standing wave method is suggested. In the measured transfer matrix, if the other three elements except T_{12} are replaced with the same constants as in Eq. (16), the matrix becomes

$$\mathbf{TM}' = \begin{bmatrix} 1 & T_{12} \\ 0 & 1 \end{bmatrix} \tag{17}$$

where the superscript prime is used here to indicate that this transfer matrix has been modified for predicting multilayer solid materials.

The essential purpose of this modified transfer matrix is to artificially make the sound velocities on both sides of the specimen the same, which accords with the real conditions of the vibration of thin plates. To predict the transmission loss of multilayer solid materials, because of the continuity of the sound pressure, and the invariance of the sound velocity, the modified transfer matrices of component layers can be multiplied together into a total matrix. To verify the feasibility of this modified transfer matrix, several experiments were implemented. Fig. 4 shows the comparison among the values of

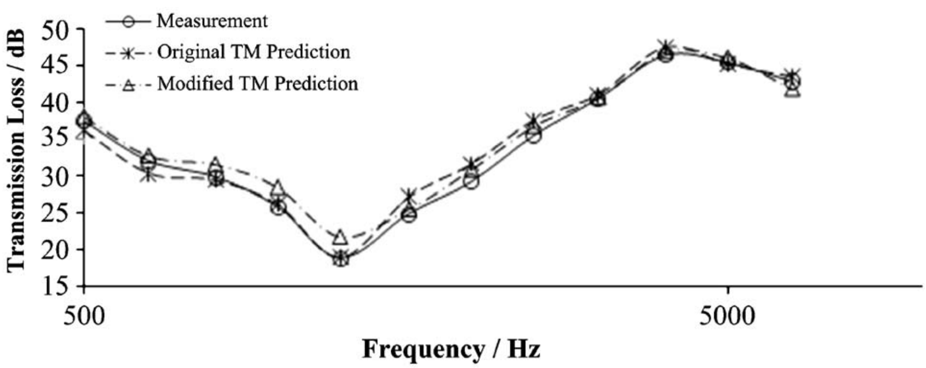

Fig. 4. Comparison among the values of measured, original transfer matrix predicted and modified transfer matrix predicted transmission loss of Specimen 6 in high frequency range.

measured, original transfer matrix predicted and modified transfer matrix predicted transmission loss of Specimen 6. Figs. 5 and 6 show comparisons between the values of measured and modified transfer matrix predicted transmission losses in multilayer specimens. It can be deduced from these figures that this modified transfer matrix can be used to predict the sound transmission loss of multilayer solid materials.

For a randomly layered multilayer material, in which both solid and porous layers exist, the modified transfer matrix is used to represent solid layers. Meanwhile the original matrix is used to represent porous layers, and the total transfer matrix is the product of all layers' transfer matrices.

$$\mathbf{TM}'_{\mathbf{Total}} = \mathbf{TM}'_{\mathbf{1}}(solid) \times \mathbf{TM}_{\mathbf{2}}(porous) \times \mathbf{TM}'_{\mathbf{3}}(solid) \cdots \mathbf{TM}_{\mathbf{n}}(porous) \quad (18)$$

From the measured transmission loss of the specimens which contain solid layers, it is obvious that the curves in the low and high frequency ranges cannot match. This phenomenon is mainly caused by the resonance of the solid layer. To get a smooth and indicative transmission loss curve, the mass law effect can be applied to amend the result.

5. Modified transfer matrix

If a limp solid panel is assumed to be unbounded, its transmission loss in a case of normal incidence can be expressed as in [11]:

$$TL_{\text{normal}}(f) = 10\,\text{lg}\left[1 + \left(\frac{\pi f m}{\rho_0 c}\right)^2\right] \quad (19)$$

where ρ_0 is the ambient fluid density, c the ambient sound speed, f the frequency of the incident sound wave and m the mass per unit area of the specimen.

In general, $\pi fm \gg \rho_0 c$, so Eq. (19) can be reduced to

$$TL_{\text{normal}}(f) = 20\,\text{lg}\left(\frac{\pi f m}{\rho_0 c}\right) = 20\,\text{lg}(fm) - 42.5 \quad (20)$$

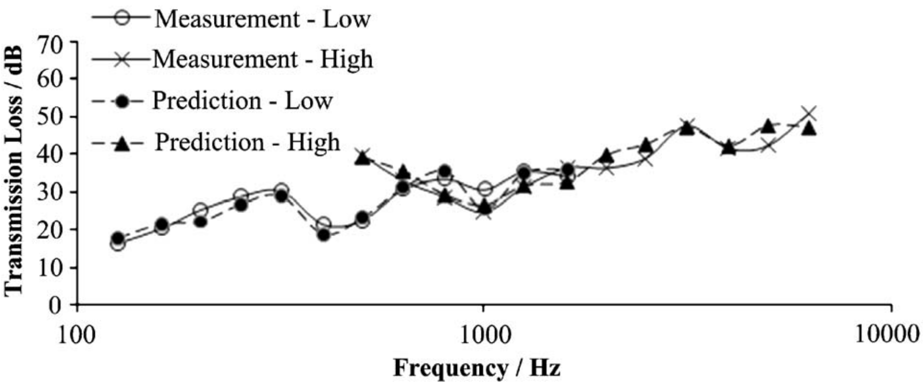

Fig. 5. Measurement and prediction results of two layer specimen (Specimens 4+5).

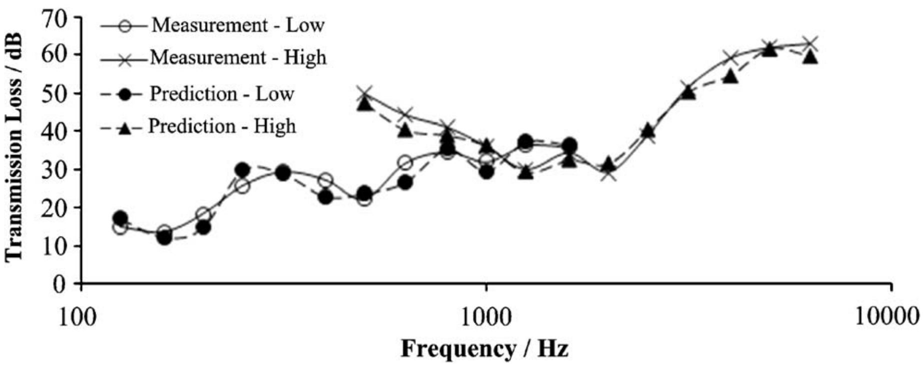

Fig. 6. Measurement and prediction results of three layer specimen (Specimens 6+4+5).

Similarly, field incidence transmission loss and normal incidence transmission loss can also be expressed by simplified equations which relate to normal incidence transmission loss [11] as

$$TL_{\text{field}}(f) = TL_{\text{normal}}(f) - 5 \tag{21}$$

$$TL_{\text{random}}(f) = TL_{\text{field}}(f) - 10\,lg(0.23 \times TL_{\text{field}}(f)) \tag{22}$$

For the transmission loss of bounded elastic solid materials measured with the reverberation room method, the phenomena of resonance and coincidence cannot be neglected because the boundary condition exists. There are three stages in the curve of transmission loss versus frequency: stiffness controlled, mass controlled and coincidence controlled, as shown in Fig. 7.

Compared with the reverberation room method, the specimen used in the standing wave tube method is much smaller and its edge is clamped tightly, so the stiffness controlled stage covers a wide range of measurable frequencies, where several resonances occur simultaneously. The resonance frequency is determined by the physical parameters and boundary conditions of the specimen, so resonances occur at different frequencies in low and high frequency standing wave tubes. In addition, the coincidence phenomenon is negligible because only the normal incidence sound propagates inside the standing wave tube. Therefore in the mass controlled regions of both low and high frequency ranges, the curves of transmission loss have the same tendency, which affirms the mass law effect. In real measurements, the mass law curve can be obtained by connecting two points which are in the low and high frequency mass law controlled range, respectively.

Fig. 8 shows how to get the mass law curve of a specimen from the measured transmission loss. In the figure, f_{01} and f_{11} are the first-order resonance frequencies in low and high frequency ranges, respectively. The measured results in the range which is higher than the resonance frequency are found to be very smooth, thus they are in the mass control range. f_{02} and f_{12} are randomly selected points in the mass control range at low and high frequencies, respectively, and their corresponding transmission loss are TL_1 and TL_2. Recalling that normal incidence mass law is determined only by frequency and the area density of the specimen, it is clear that the mass law curve approximates the line which connects f_{02} and f_{12}, and it can be expressed as

$$TL = X\,\lg(mf) - P \quad (\text{dB}) \tag{23}$$

where

$$X = (TL_2 - TL_1)/[\lg(mf_{12}) - \lg(mf_{02})] \tag{24}$$

$$P = X\,\lg(mf_{02}) - TL_1 \tag{25}$$

Fig. 9 shows an example of connecting measured transmission loss in low and high frequency ranges to get the mass law curve. The specimen is a kind of PVC synthetic leather with an area density of 0.78 kg/m^2 and a thickness of 3 mm. Two points which are in the mass law controlled range are selected, as shown in the figure, to get the mass law curve. After connecting the mass law curve, its mathematical equation is as follows:

$$TL = 19.9\,\lg(mf) - 42.8 \quad (\text{dB}) \tag{26}$$

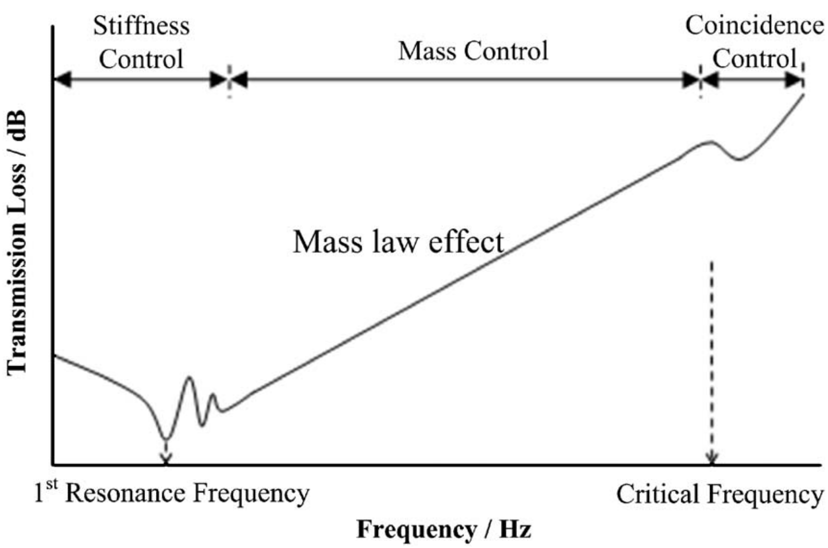

Fig. 7. Characteristic curve of sound transmission loss through an elastic solid panel.

In order to remove the resonance and other minor phenomena in the standing wave tube measured transmission loss curve, considering only the mass law effect, the transfer matrices of solid layers should be modified. With these modified matrices, not only the normal incidence transmission loss of single or multilayer materials can be predicted, but also the field or random incidence transmission loss becomes predictable.

For the sound field inside a standing wave tube, if the termination of the standing wave tube is anechoic, the reflected sound wave in the downstream section D is negligible compared to the transmitted sound wave C. The sound pressures and particle velocities on the two surfaces of the specimen can be expressed with the transmission loss as

$$P_{x=0} = A + B \tag{27}$$

$$V_{x=0} = \frac{A - B}{\rho_0 c} \tag{28}$$

$$P_{x=d} = AT\,\mathrm{e}^{-\mathrm{j}kd} = A10^{TL/20}\,\mathrm{e}^{-\mathrm{j}kd} \tag{29}$$

$$V_{x=d} = \frac{A10^{TL/20}\,\mathrm{e}^{-\mathrm{j}kd}}{\rho_0 c} \tag{30}$$

where $T = C/A$ is the transmission coefficient and TL the transmission loss calculated by Eq. (23).

To modify the transfer matrix during the test procedure, the complex numbers A and B should be recorded in the measurement, which will be used for retrieval of sound pressure and velocity at $x = 0$. At $x = d$, the values of sound pressure and velocity can be calculated by substituting the mass law dominated transmission loss, which is the result of

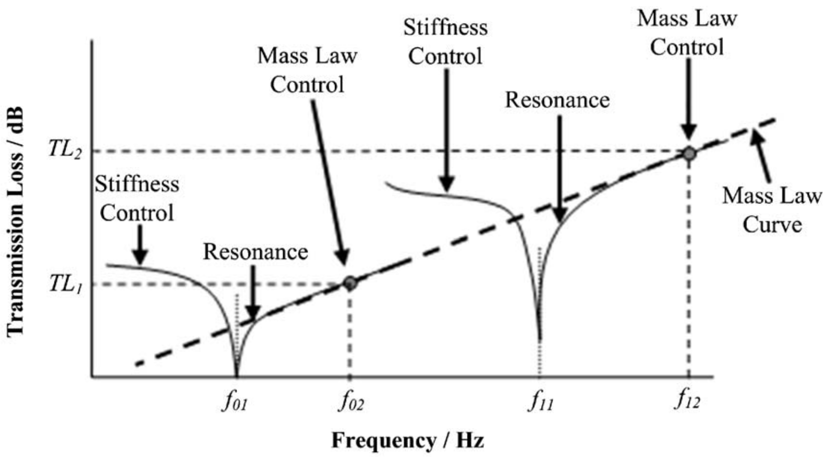

Fig. 8. Idealized curves of the transmission loss of a solid specimen tested by low and high frequency standing wave tubes.

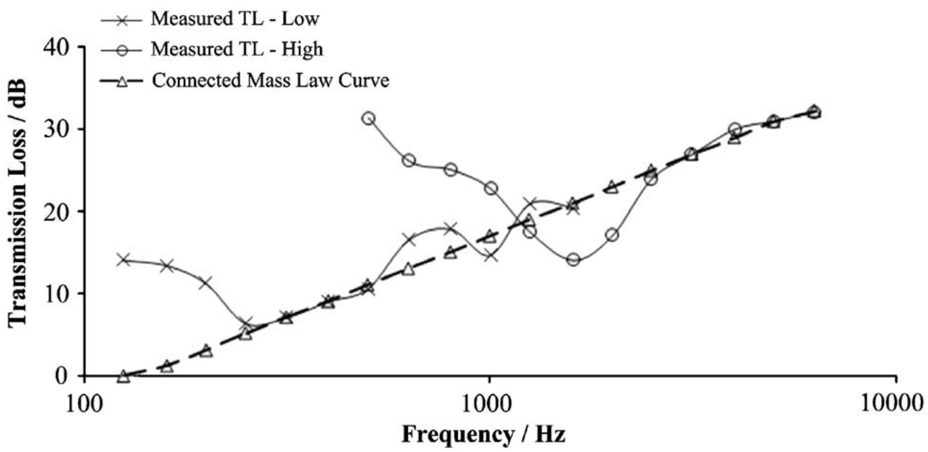

Fig. 9. Measured transmission loss of Specimen 3 and the connected mass law curve.

Eq. (23). Finally, the elements of the modified transfer matrix can be retrieved by substituting Eqs. (27)–(30) into Eqs. (6)–(9) and following the modified transfer matrix Eq. (17). The final modified transfer matrix is expressed in Eq. (31):

$$\mathbf{TM}'' = \begin{bmatrix} T''_{11} & T''_{12} \\ T''_{21} & T''_{22} \end{bmatrix} \tag{31}$$

where the superscript double prime is used here to indicate that this element has been modified by the mass law effect.

$$T''_{11} = 1 \tag{32}$$

$$T''_{12} = \frac{\rho_0 c(A^2 + B^2 + 2AB - A^2 10^{TL/10}\,\mathrm{e}^{-2jkd})}{2A^2 10^{TL/20}\,\mathrm{e}^{-jkd}} \tag{33}$$

$$T_{21} = 0 \tag{34}$$

$$T_{22} = 1 \tag{35}$$

The total transfer matrix of a multilayer material becomes

$$\mathbf{TM}''_{\mathbf{Total}} = \mathbf{TM}''_{\mathbf{1}}(solid) \times \mathbf{TM_2}(porous) \times \mathbf{TM}''_{\mathbf{3}}(solid) \cdots \mathbf{TM_n}(porous) \tag{36}$$

Fig. 10 shows the measured and predicted values of the transmission loss of Specimen 4 in log scale. The measured results are drawn with real lines, and predicted results are drawn with dashed lines. Comparing theses curves, the predicted and measured results match well in both low and high ranges of frequency. The two connected mass law curves are very close, which means that the predicted mass law curve can be used as a reference to modify the transfer matrix. The mass law curves which were connected in the same way for the three solid specimens used in this experiment are shown in Fig. 11.

After measuring and modifying all three specimens' transfer matrices, these data were recorded in the database. In the prediction of multilayer materials' transmission loss, the data of component layers was selected to build the total transfer matrix. The measured results and thereby connected mass law curves are shown in Figs. 12 and 14. To verify the modified transfer matrix method, comparisons between the predicted and the connected mass law curves are shown in Figs. 13 and 15.

From these figures (Figs. 12–15), it is clearly shown that along with the increase in the number of layers in the multilayer specimen, the transmission loss also increases, especially in the high frequency range. It is notable that transmission loss does not change too much in the low frequency range, which tells us that multilayer materials have an ameliorative sound insulation ability in the high frequency range. Exactly speaking, these specimens were not associated with each other during the measurement. In fact, each layer's own characteristics exist in multilayer specimens' transmission loss curves, which is why more than one resonance occurs in the high frequency range. Although resonance phenomena become more complicated for multilayer specimens compared with the single layer case, the predicted mass law curves match well with the curves which are connected by real results.

In the experiment, a sandwich specimen was assembled using two solid specimens (Specimens 4 and 5) as skins, and one porous specimen (Specimen 1) as the core. When predicting the transmission loss of this sandwich specimen, the

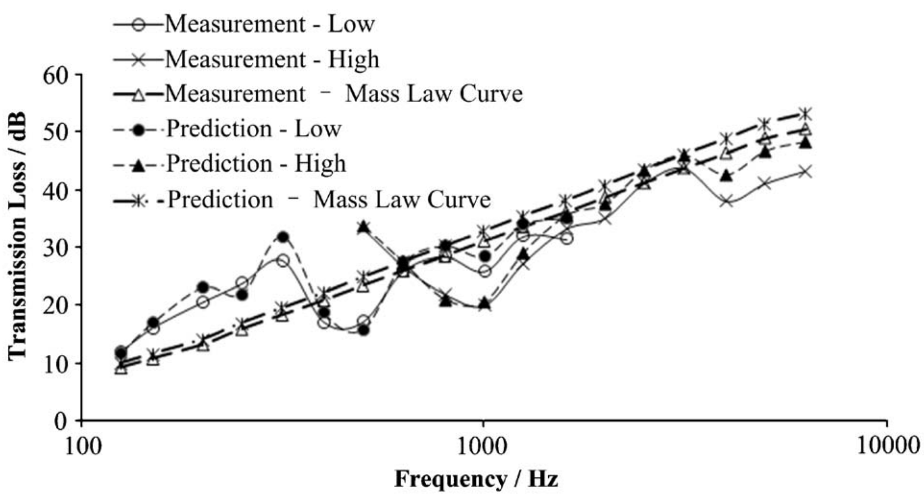

Fig. 10. Tested and predicted results of transmission loss of Specimen 4.

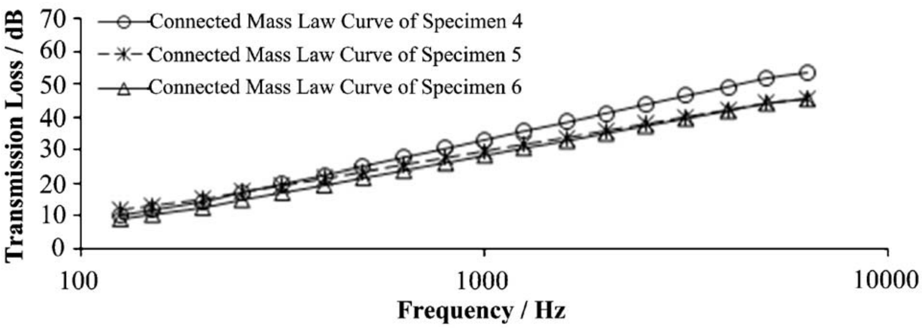

Fig. 11. Connected mass law curves of the specimens.

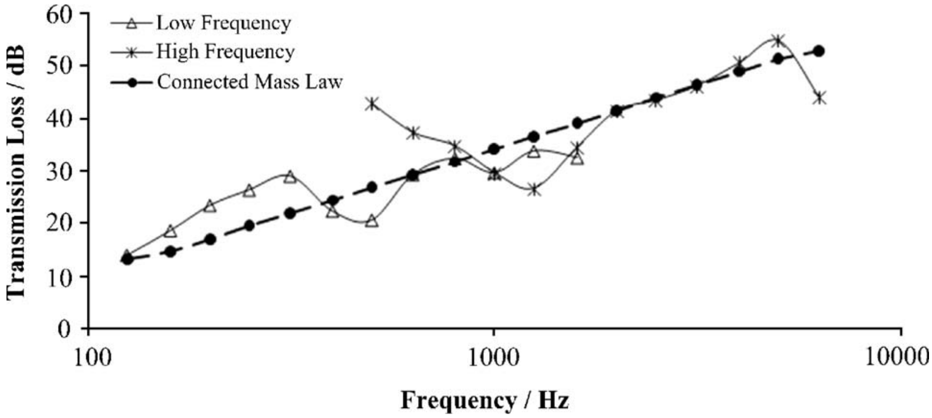

Fig. 12. Measured results and connected mass law curve of Specimens 4+5.

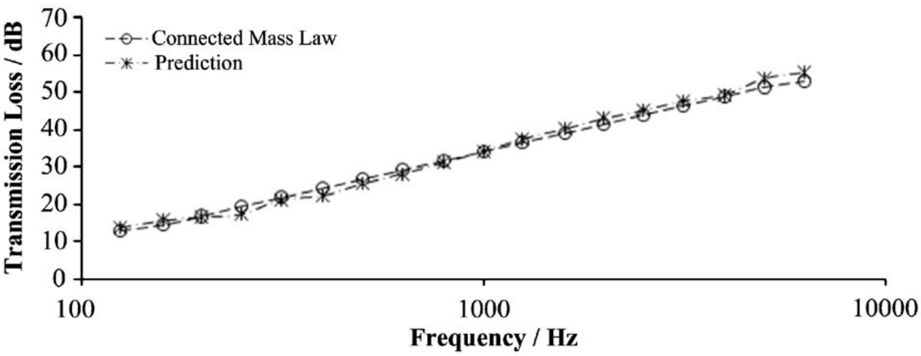

Fig. 13. Comparison between predicted and measured mass law curves of Specimens 4+5.

modified transfer matrices were used to represent the two solid specimens, and the original transfer matrix was used to represent the porous specimen. Fig. 16 shows the measured and predicted transmission loss of this sandwich specimen.

Because of the porous layer, the transmission loss curve of this assembled sandwich specimen was much more complicated compared with simple, multilayer solid specimens, because there was no exact mass controlled range. The original measured transfer matrix was used to present this porous layer, so the prediction was very close to the measured result.

6. Conclusion

Our purpose was to develop a convenient method for evaluating the sound transmission losses that are characteristic of commonly used multilayer acoustic insulation materials. The procedure we present was based on the well-known transfer matrix method, but modified for solid layers, by taking advantage of thin plate theory and the mass law effect. For both single layer and multilayer materials, the results predicted using the modified transfer matrix approach were comparable to the direct measurements in the mass law controlled range, which supports the validity and applicability of our new method.

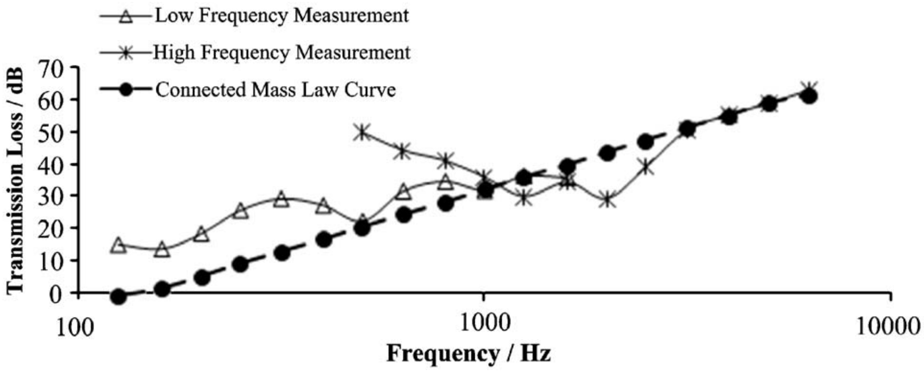

Fig. 14. Measured results and connected mass law curve of Specimens 4+5+6.

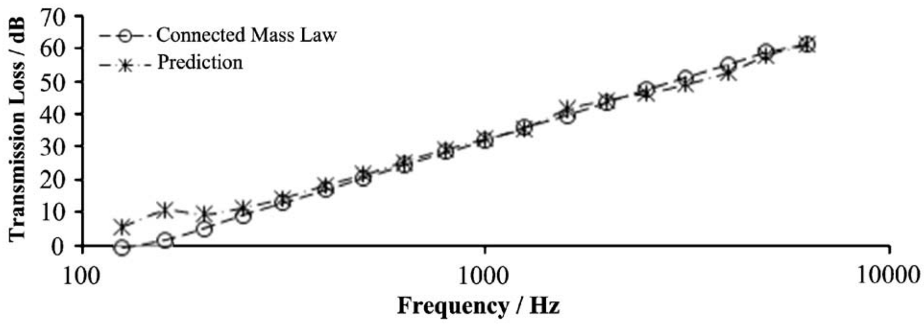

Fig. 15. Comparison between predicted and measured and mass law curves of Specimens 4+5+6.

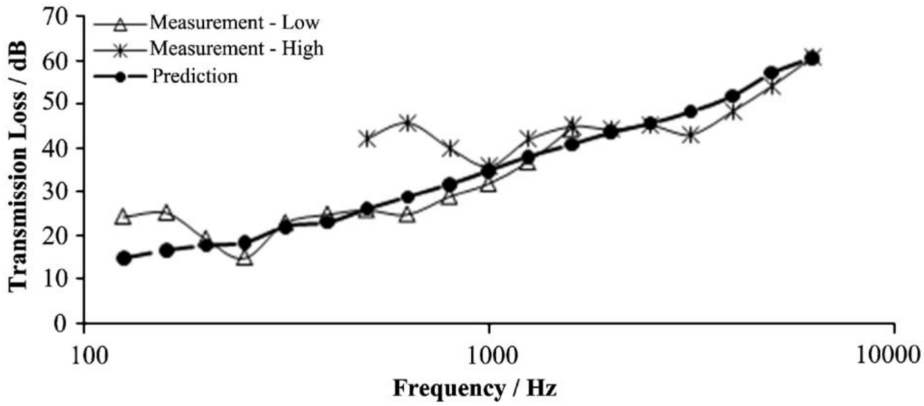

Fig. 16. Measurement and prediction results of the sandwich specimen.

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Appendix A. Four-microphone standing wave ratio method

In this paper, a four-microphone standing wave ratio method is applied to obtain the standard measured transmission loss of a specimen. As the schematic diagram shown in Fig. 1, in the upstream section of a standing wave tube, two microphones (1 and 2) are used to measure the sound pressures at different locations. The auto-spectral densities S_{11} and S_{22} of the two signals and their cross-spectral density $S_{12} = R_{12}+jI_{12}$ are captured and used to calculate the separated auto-spectra S_{AA} and S_{BB} of the incident and reflected sound waves.

$$\begin{bmatrix} S_{AA} \\ S_{BB} \\ R_{AB} \\ I_{AB} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 2\cos 2kx_1 & 2\sin 2kx_1 \\ 1 & 1 & 2\cos 2kx_2 & 2\sin 2kx_2 \\ \cos kn_1 & \cos kn_1 & 2\cos km_1 & 2\sin km_1 \\ -\sin kn_1 & \sin kn_1 & 0 & 0 \end{bmatrix} \begin{bmatrix} S_{11} \\ S_{22} \\ R_{12} \\ I_{12} \end{bmatrix} \tag{A.1}$$

where $S_{AB} = R_{AB}$+jQ_{AB} is the cross-spectrum between the incident and reflected waves, x_1 and x_2 represent the distances from the front surface of the specimen to microphones 1 and 2, respectively, and $m_1 = x_1+x_2$, $n_1 = x_1-x_2$.

Similarly, the auto-spectrum S_{CC} in the downstream section may be obtained by imposing the same computations on the second pair of microphones. Note that the S_{DD} should approach zero due to the anechoic termination.

$$\begin{bmatrix} S_{CC} \\ S_{DD} \\ R_{CD} \\ I_{CD} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 2\cos 2kx_3 & 2\sin 2kx_3 \\ 1 & 1 & 2\cos 2kx_4 & 2\sin 2kx_4 \\ \cos kn_2 & \cos kn_2 & 2\cos km_2 & 2\sin km_2 \\ -\sin kn_2 & \sin kn_2 & 0 & 0 \end{bmatrix} \begin{bmatrix} S_{33} \\ S_{44} \\ R_{34} \\ I_{34} \end{bmatrix} \tag{A.2}$$

where x_3 and x_4 represent the distances from the front surface of the specimen to microphones 3 and 4, respectively, and $m_2 = x_3+x_4$, $n_2 = x_3-x_4$.

According to the definition of transmission loss, the transmission loss is calculated like this:

$$TL = 10\lg\left(\frac{S_{AA}}{S_{CC}}\right) \tag{A.3}$$

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